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 Differentiate the following with respect to x
 5, $\frac{e^{4x} \sin x}{x \cos 2x}$

Sol

Let $y = \frac{e^{4x} \sin x}{x \cos 2x}$

Let $u = e^{4x} \sin x$

from u , let $t = e^{4x}$, $\frac{dt}{dx} = 4e^{4x}$
 $s = \sin x$ $\frac{ds}{dx} = \cos x$

$$\frac{du}{dx} = s \frac{dt}{dx} + t \frac{ds}{dx}$$

$$\frac{du}{dx} = \sin x (4e^{4x}) + e^{4x} (\cos x)$$

$$= 4e^{4x} \sin x + e^{4x} \cos x$$

$$\frac{du}{dx} = e^{4x} (4 \sin x + \cos x)$$

Let $V = x \cos 2x$

from V , let $P = x$, $\frac{dP}{dx} = 1$
 $w = \cos 2x$ $\frac{dw}{dx} = -2 \sin 2x$

$$\frac{dv}{dx} = w \frac{dP}{dx} + P \frac{dw}{dx}$$

$$= \cos 2x (1) + x (-2 \sin 2x)$$

$$\frac{dv}{dx} = \cos 2x - 2x \sin 2x$$

$$\frac{dy}{dx} = \frac{V \frac{du}{dx} - u \frac{dv}{dx}}{V^2}$$

$$\frac{dy}{dx} = \frac{x \cos 2x [e^{4x} (4 \sin x + \cos x)] - e^{4x} \sin x (\cos 2x - 2x \sin 2x)}{(x \cos 2x)^2}$$

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$$\frac{dy}{dx} = \frac{x \cos 2x (4e^{4x} \sin x + \cos x) - e^{4x} \sin x (\cos 2x - 2x \sin 2x)}{x^2 \cos^4 4x^2}$$

$$\frac{dy}{dx} = \frac{4x e^{4x} \sin x \cos 2x + x \cos^2 2x^2 - e^{4x} \sin x \cos 2x + 2x e^{4x} \sin 2x}{x^2 \cos^4 4x^2}$$

$$\frac{dy}{dx} = \frac{e^{4x} \sin x \cos 2x (4x - 1) + x \cos^2 2x^2 + 2x e^{4x} \sin 2x}{x^2 \cos^4 4x^2}$$

$$\frac{dy}{dx} = \frac{e^{4x} \sin x \cos 2x (4x - 1) + x (\cos^2 2x^2 + 2x e^{4x} \sin 2x)}{x^2 \cos^4 4x^2}$$

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6. $\frac{x^4}{(x+1)^2}$

Let $u = x^4$, $\frac{du}{dx} = 4x^3$

$v = (x+1)^2$, $\frac{dv}{dx} = 2 \cdot (x+1)^{2-1} \cdot 1 = 2(x+1) = 2x+2$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)^2 (4x^3) - x^4 (2x+2)}{(x+1)^4}$$

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$$= \frac{4x^3 (x+1)^2 - x^4 (2x+2)}{(x+1)^4}$$

$$= \frac{4x^3 (x^2 + 2x + 1) - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 + 8x^3 + 4x^3 - 2x^5 - 2x^4}{(x+1)^4}$$

$$= \frac{4x^5 - 2x^5 - 2x^4 + 12x^3}{(x+1)^4}$$

$$= \frac{2x^5 - 2x^4 + 12x^3}{(x+1)^4} \quad \text{OR} \quad \frac{2x^5 - 2x^4 + 12x^3}{3x^4 + 2x^3 + 6x^2 + 4x + 1}$$

$$6. y = \ln(3-4\cos x)$$

Sol

$$y = \ln(3-4\cos x)$$

$$\text{let } u = 3-4\cos x$$

$$\frac{dy}{dx} = \frac{1}{3-4\cos x}$$

$$y = \ln u$$

$$\frac{dy}{dx} = \frac{1}{u} \cdot u'$$

$$\frac{dy}{dx} = \frac{1}{3-4\cos x} \cdot (3-4\cos x)'$$

$$\frac{dy}{dx} = \frac{4\sin x}{3-4\cos x}$$

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$$2. \text{ If } x^2 + y^3 - 4x + 4y = 26 \text{ find } \frac{dy}{dx} \text{ and } \frac{d^2y}{dx^2}$$

Sol

$$x^2 + y^3 - 4x + 4y = 26$$

$$2x + 3y^2 \frac{dy}{dx} - 4 + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (3y^2 + 4) = 4 - 2x$$

$$\frac{dy}{dx} = \frac{4-2x}{3y^2+4}$$

To get $\frac{d^2y}{dx^2}$

$$\text{let } u = 4-2x$$

$$v = 3y^2 + 4$$

$$\frac{du}{dx} = -2$$

$$\frac{dv}{dx} = 6y \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{(3y^2+4)(-2) - (4-2x)(6y \frac{dy}{dx})}{(3y^2+4)^2}$$

$$= \frac{(-6y^2-8) - (24y \frac{dy}{dx} - 12xy \frac{dy}{dx})}{(3y^2+4)^2}$$

$$= \frac{(-6y^2-8) - 24y \frac{dy}{dx} + 12xy \frac{dy}{dx}}{(3y^2+4)^2}$$

$$= \frac{-6y^2-8 - (24y - 12xy) \frac{dy}{dx}}{(3y^2+4)^2}$$

$$= \frac{-6y^2-8 - (24y - 12xy) \left(\frac{4-2x}{3y^2+4} \right)}{(3y^2+4)^2}$$

$$= \frac{-6y^2-8 - \left[\frac{96y-48yx}{3y^2+4} - \frac{48xy-24x^2y}{3y^2+4} \right]}{(3y^2+4)^2}$$

$$= \frac{-6y^2-8 - \left[\frac{96y-48yx-48xy+24x^2y}{3y^2+4} \right]}{(3y^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-6y^2 - 8 - 96y + 48yx + 48xy - 24x^2y}{3y^2 + 4}$$

$$\frac{d^2y}{dx^2} = \frac{3y^2 + 4(-6y^2 - 8) - 96y + 48yx + 48xy - 24x^2y}{(3y^2 + 4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{3y^2 + 4(-6y^2 - 8) - 96y + 96xy - 24x^2y}{3y^2 + 4} \times \frac{(3y^2 + 4)^2}{1}$$

$$= (3y^2 + 4)^2 (-6y^2 - 8) - (96y + 96xy - 24x^2y)(3y^2 + 4)$$

$$\frac{d^2y}{dx^2} = 3y^2 + 4 [3y^2 + 4(-6y^2 - 8) - 96y - 96xy + 24x^2y]$$

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